services. And only in this case is it economically meaningful to measure separate TFPs for the two services.

7. In the application of his Performance-Based Model to the measurement of interstate access TFP, Norsworthy ignores this well-established principle that costs incurred jointly by two or more services cannot be separated in an economically meaningful way[3]. In fact he claims that "no specific allocation of costs is required" (page 27). If this were true we would have the productivity analyst's version of the free lunch. It would mean that TFP for interstate access services could be calculated without knowing anything about the inputs used to produce these services!

8. Unfortunately, the free lunch is an illusion. The statement above is immediately followed by Norsworthy's cost allocation assumption that "inputs grow at the same rates for all classes of services." (page 27). This is a particularly simplistic form of cost allocation which cannot be taken seriously as an economically meaningful allocation. Norsworthy's reliance on such a simplistic form of cost allocation is an implicit admission that no economically meaningful cost allocation procedure to determine the "costs" of interstate access services is possible.

9. Norsworthy is also incorrect in his claim that no specific allocation of costs is required by him, since he is only making an assumption about the growth rates of inputs used to produce specific services, not their levels. It is obvious that the assumption used by Norsworthy imposes a specific (i.e, equal percentage) allocation of

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[3] This principle is not ignored by Christensen. No attempt is made in the USTA model to obtain an interstate-specific TFP measure. Christensen's procedure is conceptually correct.

input **changes**. In any productivity study the initial (for example, 1984) levels of all inputs can be chosen arbitrarily, and are often set to unity. This must be the case otherwise the TFP estimates would be sensitive to the units of measurement of the inputs. Since the initial levels of inputs are arbitrary, allocating the changes in inputs is tantamount to allocating all the meaningful information about the inputs. It is in this sense that using the assumption of equal growth rates of inputs is equivalent to employing a specific fully-distributed cost allocation procedure.

10. A direct consequence of Norsworthy's cost allocation procedure (inputs grow at the same rate for all services) is that since the outputs for interstate access services are growing more rapidly than those for intrastate services, a higher TFP offset is assigned to these faster growing services.[4] This assignment is as arbitrary and lacking in economic meaning as the cost allocation procedure which determines it.

3. The Input Price Differential

11. Aside from repeating the conclusions of Bush and Uretsky's Appendix F regarding the input price differential, Norsworthy offers the following additional arguments:

(i) A chi-squared test of the equality of the annual LEC and US input price changes using the Christensen data.

(ii) His own measurement of the input price differential.

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[4] It is more than a direct consequence, it is an algebraic truism.

These two additional arguments are not persuasive. The first argument is based on a test which is so stringent it is not meaningful, since to pass the test the two input price series would have to be virtually identical in every year[5]. The second argument is based on LEC input price data generated from Norsworthy's Performance-Based Model. This model is sufficiently flawed that the price data it generates are not reliable.

Problems With Norsworthy's Performance-Based Model

12. Norsworthy has provided his own measurement of the LEC's input price growth rate for the period 1985-94, and hence his own measurement of the input price differential. There are a number of significant problems with his capital data construction procedures which render the capital input price series unreliable. In addition, Norsworthy's use of a quality-adjusted capital input price series makes his calculated input price differential non-comparable to the Bush-Uretsky result, even though on the surface they appear to be of similar magnitude.

13. There are at least six problems with the capital data (input and input price) which render the calculated input price differential unreliable.

(a). First, Norsworthy uses the book value of the capital stock as his measure of the capital input. Book value is an accounting concept which generally bears little relationship to the economic value of capital. Replacement value is the conceptually correct measure of the economic value of capital. I know of no productivity analyst

[5] The extreme stringency of this test is explored in Appendix B.

who would use book value of capital without apologizing for the inability to construct estimates of replacement value due to the lack of data. The Christensen simplified TFP model constructs replacement value of capital based on the available US government price deflators.

(b). Second, use of **net** book value implies the use of FCC prescribed depreciation rates which are unlikely to mirror the economic depreciation rates needed to construct economically relevant net stocks of capital.

(c). Third, the use of book value of capital and FCC prescribed depreciation rates means that Norsworthy's calculated rate of return is an accounting rate of return. His accounting rate of return is unlikely to correspond to the economic rate of return which is the correct rate of return to include in the calculation of the capital input price. To determine the correct economic rate of return, Norsworthy would need to use the economic value of capital and economic depreciation rates in his calculation. The economic value of the capital stock and economic depreciation rates are used in Christensen's TFP model.

(d). Fourth, while both debt and equity sources of financing should be taken into account in the construction of the user cost of capital (as the Christensen simplified approach does), Norsworthy's treatment appears to be incorrect. Judging from his calculation of the alleged overstatement of the user cost of capital in Christensen's earlier versions of his productivity model (page 82), Norsworthy appears to be assuming that the pre-tax cost of debt is equal to the post-tax cost of equity. Debt and equity embody different risk factors due to a firm's different obligations to the holders of these two sources of capital financing. There is no apparent reason why the costs of debt and equity should be equal[6] .

(e). Fifth, Norsworthy's reformulation of the equation for calculating the capital input price in order to account for the unequal taxation treatment of debt and equity costs appears to be incorrect[7]. Since the derivation of the tax-adjusted formula is not given, a more definitive statement is not possible. But his equation is inconsistent with previous versions of the user cost of capital services which treat tax effects explicitly, such as those found in Fuss and Waverman (1981), and Jorgenson and Yun (1991).

(f). Sixth, Norsworthy's method of quality adjustment is flawed. He adjusts the capital input price for quality changes using procedures based on his previous research (Norsworthy et al (1993))[8]. The econometric model which underlies this adjustment contains unwarranted parameter restrictions. While the details are fairly technical,[9] in essence what has happened is that Norsworthy has imposed more restrictions on the

[6] Carlton and Perloff (1994, page 337) provide a textbook statement of the importance of recognizing the differing costs of debt and equity capital in the calculation of the firm's rate of return.

[7] The formula is given in equation (14) of Attachment 2, Appendix A of Norsworthy's Statement.

[8] These quality adjustment procedures do not correspond to the standard hedonic methodology. The hedonic methodology analyses the relationships between the prices (or production costs) of capital equipment and the quality characteristics of the equipment.

[9] It is common in cost function estimation to impose those parameter restrictions necessary to ensure that the cost function is linearly homogeneous in input prices. This will ensure that the function being estimated has the property that if all input prices increased by some percentage, total cost would increase by the same percentage. Norsworthy imposes these parameter restrictions, but also imposes additional parameter restrictions which are not needed to ensure that the estimated cost function satisfies this property.

parameters of his model than are necessary, thus biasing in unknown ways the estimates of those parameters that determine the quality adjustments.

14. The model is also specified in such a way that **all** technological progress in the supply of telecommunication services associated with reductions in labor and material costs results in an increase in the quality of capital inputs. This increased quality appears as "more" capital input in the TFP calculations. In effect, all technological progress which results in savings in labor and material costs is credited to the equipment supplying industry, and none is credited to the industry which supplies telecommunication services.

15. Due to the problem identified in the last paragraph, Norsworthy's model specification imparts an upward bias to the capital input quality adjustment, which implies an upward bias in the capital input growth rate. This upward bias has an important effect on the input price differential. Because there is an upward bias in the growth rate of the capital input quantity series, there is a corresponding downward bias in the growth rate of the capital input price series. The result is that Norsworthy's input price differential is overstated.

16. However, since the resulting downward adjustment in the rate of change of the capital input price is accompanied by a corresponding and offsetting upward adjustment in the rate of change of the capital input, Norsworthy's flawed quality adjustment procedure has no impact on his calculated X factor. Therefore Norsworthy's criticism

that Christensen does not explicitly adjust his capital data for quality change is of no relevance from a quantitative perspective.[10]

17. Norsworthy fails to point out that his calculated input price differential of 2.54% is not comparable to the 2.23% calculated by Bush and Uretsky. In fact the implication is left with the reader that the closeness of the two numbers is confirmation of their validity. In fact, Norsworthy's input price differential estimate contains a quality adjustment whereas Bush and Uretsky's does not; the two numbers are simply not comparable, and their apparent closeness is simply an irrelevant coincidence[11].

18. These two differentials can be placed on a more comparable basis by removing the quality adjustment from Norsworthy's data. On page 34, Norsworthy states that his quality adjustment resulted in "a very small (or no) effect on the X factor". Since,

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[10] The fact that quality change adjustments have no effect on Norsworthy's estimate of the X factor is peculiar to his adjustment procedure. In general, quality adjustment using hedonics can increase or decrease the X factor; a zero effect is unlikely unless the economy-wide TFP and input price changes are adjusted as well - something that Norsworthy did not do.

It is very difficult to adjust the X factor properly for quality change, even given agreement on the econometric equation which provides the quality estimates. While the capital input price growth rate can be adjusted directly for quality change, proper quality adjustment of the capital input quantity growth rate is another matter. The reason why quality adjustment of the capital input growth rate is problematic is that at any point in time all surviving capital would need to be adjusted. This would require a complete historical knowledge of the characteristics of the past capital put in place which comprises the benchmark capital.

This problem carries through to ETI's comments on behalf of Ad Hoc. In all likelihood because of the difficulties in making quality adjustment to capital growth, ETI does not adjust Christensen's benchmark capital stock for quality change in their quality adjustment simulations. But this omission results in an incorrect procedure which biases upward ETI's estimated X factor. The input quantity growth rate is biased downward, but the input price growth rate does not contain an offsetting bias (as is the case with Norsworthy's calculations).

ETI's capital input growth rate is underestimated because the failure to quality-adjust the benchmark stock renders it too large, and hence too much of the observed investment over the 1984-93 period is replacement investment, rather than additions to the net capital stock. Under the perpetual inventory capital accumulation methodology, when replacement investment is too large, the net capital stock grows too slowly.

[11] It should be noted that Norsworthy only adjusts the LECs input prices for quality change. In order to be consistent, he should also adjust the US economy input prices for quality change.

from table 15, we can calculate that the quality adjustment increased the growth rate of the capital input by 3.27%, the growth rate of the capital input price used in the LEC input price series must have decreased by approximately 3.27%, if the X-factor is to remain virtually unchanged. Replacing the adjusted capital input price index with the unadjusted one in table 5, and assuming that capital costs are 50% of total costs, the unadjusted input price growth rate differential is 0.91%. This is the number which should be compared with Bush and Uretsky's 2.23%. Regardless, neither calculation provides a legitimate basis to conclude that there should be an ongoing non-zero input price differential.

4. Depreciation Rates

19. The depreciation of capital in the Christensen model is based on economic depreciation rates as calculated using the Hulten and Wykoff (1981) methodology. The Hulten-Wykoff methodology is used in most productivity studies which calculate capital accumulation using the perpetual inventory procedure. This is because their methodology accounts for both obsolescence (which leads to retirements which are not observed) and physical decay (which leads to reduced efficiency). Hence the Hulten-Wykoff methodology estimates the economic depreciation of capital. It is certainly true that the depreciation rates calculated by Hulten and Wykoff are likely to be dated. However, the Christensen model does not use the Hulten-Wykoff depreciation rates, but rather Hulten and Wykoff's estimated relationships between economic lifetimes and

geometric depreciation rates[12]. The Christensen procedure is to substitute the most recent Bureau of Economic Analysis lifetimes into these relationships to obtain depreciation rates. While it is clear that economic lifetimes have changed since the 1970s (they have surely shortened for telecommunications equipment), it is not at all clear that the relationship between depreciation rates and lifetimes have changed. This is an empirical question, and I agree with Norsworthy that this question is worthy of study. It is not, however, a feasible recommendation for the current proceedings. Such a study would be a long term research project, since it would be necessary to collect and analyse a large volume of transaction prices of used telecommunications equipment.

20. Finally, I find it strange that Norsworthy criticizes the Christensen depreciation rates as being **too high** (page 49) (emphasis mine). If anything, the Bureau of Economic Analysis communications equipment lifetimes will overstate the actual lifetimes due to the probable increasing importance of obsolescence which has not as yet found its way into official government lifetime estimates[13].

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[12] As both Norsworthy and ETI remark, the estimated relationships, involving a factor of 1.65 for communications equipment and 0.90 for telephone and telegraph structures, are based on averages of other classes of capital. It should be noted that these averages were chosen as reasonable by the two leading researchers in the area of estimating economic depreciation rates.

[13] ETI provides estimates of TFP growth when the depreciation rates are increased relative to those used in the Christensen model. However ETI's failure to adjust Christensen's benchmark capital stock for the assumption of increased depreciation rates means that their estimates of TFP growth (and the X factor) are biased upwards. Similar to the problem ETI encounters with its quality adjustment procedure, ETI's capital input growth rate is underestimated, and hence TFP growth is overestimated. This occurs because the failure to adjust the benchmark stock for the increased depreciation rate renders it too large. Too much of the observed investment over the 1984-93 period is replacement investment, rather than additions to the net capital stock. Under the perpetual inventory capital accumulation methodology, when replacement investment is too large, the net capital stock grows too slowly.

5. Revenue Weights vs Marginal Cost Weights in Price Caps Formulas

21. On pages 59-62, Norsworthy argues that the Christensen model violates the economic theory of production because it erroneously uses revenue weights rather than marginal cost weights in the construction of the productivity offset. He asserts that marginal cost-weighted output growth should necessarily replace revenue-weighted output growth in the Christensen calculations. Norsworthy is incorrect in his assertion. The issue is considerably more complex than Norsworthy recognizes.

22. The revenue weighted TFP measure cannot simply be replaced by a marginal cost weighted measure, as Norsworthy appears to believe. While it is possible to use either revenue-weighted or cost-weighted output growth in the price caps formula, using cost-weighted output growth adds a degree of complexity to the formula. The tradeoffs involved are explored in Appendix C. Given the current state of our knowledge about the magnitudes of the marginal cost weights in U.S. telecommunications, I prefer Christensen's use of revenue-weights in the construction of TFP growth for price caps purposes. Christensen's methodology is certainly superior to Norsworthy's incorrect use of his revenue requirements cost weights proxies.

6. Fisher vs Tornquist Indexing Methods

23. Norsworthy makes too much of the difference between using the Tornquist index (as the Christensen model does) and using the Fisher Ideal Index (as his model

does) to aggregate prices and quantities. From an empirical perspective it usually makes little difference which index is used.

24. Table 1 demonstrates the close correspondence between the Fisher and Tornquist indices for the data contained in Norsworthy's model. I have used Norsworthy's data to construct aggregate output and input Fisher and Tornquist indices for the LECs' regulated services[14]. I have also constructed these indices for the aggregate input price[15]. As can be seen from table 1, the growth rates of the Fisher and Tornquist indices of aggregate output, input, and input price are virtually identical. The yearly indices are presented in table D.1 of Appendix D. The Fisher and Tornquist indices are almost identical, to three decimal places.

25. The close correspondence between the Fisher and Tornquist indices which I have found is not surprising and has been noted in the literature. Appendix D contains a detailed description of the rationale behind this close correspondence.

6. Conclusion

26. AT&T and Ad Hoc's central criticisms of the Christensen productivity model are unfounded. Indeed, it is AT&T's proposed alternative model that contains fundamental economic flaws.

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[14] Norsworthy's Fisher Index growth rate calculations appear in table 7.

[15] These indices correspond to the data in Norsworthy's table 5.

I declare, under penalty of perjury, that the foregoing is true and correct.

Executed on February 26, 1996

Melvyn A. Fuss

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Table 1

A Comparison of Fisher and Tornquist Index Growth Rates

Norsworthy Data 1985-94

Aggregation	Fisher Index	Tornquist Index
Output Growth - All LEC Regulated Services	4.90%	4.90%
Input Growth - All LEC Regulated Services	1.89%	1.90%
Input Price Growth - All Inputs	0.46%	0.46%

Appendix A

The Impossibility of Calculating Service-Specific TFP for Interstate Access

It is well known in the theory of production literature that when outputs use factors of production in common, separate production functions cannot be defined (see Hall (1973), Denny and Pinto (1978, Chambers (1988)). Since interstate services use factors of production in common with other services, a separate production function cannot be defined for interstate services.

To be more specific, suppose the telecommunications firm produces two outputs: interstate (Y_1) and intrastate (Y_2), using two inputs X_1 and X_2. Then the production process for the multiproduct firm can be written as

$$F[Y_1,Y_2,X_1,X_2] = 0 \tag{1}$$

In the special case where the production function is separable, (1) can be written in the form

$$F[Y_1(X_1,X_2),Y_2(X_1,X_2)] = 0 \tag{2}$$

where $Y_1(X_1,X_2)$ and $Y_2(X_1,X_2)$ are separate production functions for the production of Y_1 and Y_2 respectively.[16]

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[16] From a technical perspective, (2) implies that the marginal rate of substitution between the two inputs in the production of Y_1 does not depend on the level of Y_2, and vice versa.

Equations (1) and (2) can also be written in terms of cost functions. Suppose p_1 and p_2 are the factor prices for the two inputs. Then the cost function for the general production function (1) can be written as

$$C = C(p_1,p_2,Y_1,Y_2) \tag{3}$$

where C is the firm's total cost of producing the two outputs. In the special case where the production function is separable (equation (2)), the cost function can be written in the form

$$C = C_1(p_1,p_2,Y_1) + C_2(p_1,p_2,Y_2) \tag{4}$$

where C_1 and C_2 are the costs of producing output 1 and output 2 respectively. Only in the case where the cost function can be written in the form (4) can total costs be **separated** in any economically meaningful way into the costs of producing output 1 and the costs of producing output 2. And only in this case is it economically meaningful to measure separate TFPs for the two services.

The cost function can only be written in the form (4) if there are no economies of scope in the production of the two outputs. Economies of scope result when inputs are used in common by the two services. Clearly this is the case in telecommunications, where a substantial amount of labor and especially capital are jointly used by interstate and intrastate services. As noted earlier, there is no economically meaningful way to separate the costs of such jointly used inputs in order to calculate service-specific TFP growth.[17]

[17] The only efficiency-related concept which can be calculated at the level of a specific service is the decline in marginal cost which may occur when outputs grow. Growth in interstate output can lead to a decline in the marginal cost of producing interstate services (and hence a decline in the interstate price required by the firm) through economies of scale. But this growth can also lead to a decline in the marginal cost of intrastate services (and hence a decline in the intrastate price required by the firm) through economies of scope. Similarly, growth in intrastate output can lead to declines in the marginal costs of both interstate and intrastate services. The relative impacts of economies of scale and economies of scope on the prices required by the firm over time cannot be determined unless a detailed knowledge of the joint cost function (equation (3) in Appendix A) is available. Even if such knowledge were available, service-specific TFPs still could not be calculated since they do not exist conceptually. A single firm-wide TFP estimate would still be the only economically meaningful productivity offset in a price-caps formula.

Appendix B

The Stringency of Norsworthy's Chi-squared Test of the Input Price Differential

Consider the proposed chi-squared test of the equality of the LEC and US input price changes (page 8 of Norsworthy's Statement). Instead of testing whether the two series differ on average over a specific period of time by a random variable with mean zero, Norsworthy proposes a test which tests whether the two series are identical **in each year**. To see the peculiar implications of this test, suppose all parties could agree that the relevant time period for measuring the average input price differential for inclusion in a price caps formula was the period 1949-92. From Norsworthy's table 1, the average increase in the LEC's input prices over this period was 4.70%, and the average increase in the US economy's input prices was 4.75%. It is hard to imagine that a differential of 0.05% would spark much of a debate as to whether or not an average input price differential should be included in the price caps formula. Yet the probability that the two series are the same is an infinitesimal 0.0000000001! Clearly, the chi-squared test proposed by Norsworthy is not relevant to the question of whether a fixed average input price differential should be included in the price caps formula.

Appendix C

Revenue Weights vs Marginal Cost Weights in Price Caps Formulas

The output price index growth rate of a firm, as a matter of algebra, takes the form

$$\text{Output Price Index} = \text{Input Price Index} - TFP^{R} \quad (5)$$

where TFP^R is the revenue-weighted measure of TFP growth (i.e., outputs are aggregated using revenue weights). Denny, Fuss and Waverman (1981) demonstrated that TFP^R could be expressed as

$$TFP^{R} = TFP^{C} + (Y^{R} - Y^{C}) \quad (6)$$

where TFP^C is the marginal cost-weighted measure of TFP growth, Y^R is the growth rate of aggregate output when revenue weights are used in the aggregation formula, and Y^C is the growth rate of aggregate output when marginal cost weights are used in the aggregation formula.

Equation (6) implies that an alternative expression for the output price index growth rate is

$$\text{Output Price Index} = \text{Input Price Index} - TFP^{C} - (Y^{R} - Y^{C}) \quad (7)$$

As I have recently emphasized (Fuss (1994)), the revenue-weighted measure of TFP growth (TFP^R) only represents efficiency growth if output prices are proportional to marginal costs or the rates of change of all outputs are equal. If at least one of these conditions is not met, efficiency change is more accurately represented by a marginal cost-weighted measure of TFP growth (TFP^C).

While it is clear that when revenue-weighted and cost-weighted TFP growth rates differ historically, the cost-weighted measure is a superior indicator of past efficiency growth, it is not as clear which measure should be used in determining the productivity offset in a price-caps formula. This is because the productivity offset in a price caps formula measures more than efficiency changes; it measures the ability of the firm to sustain output price declines, net of inflation. So, for example, if intensified competition causes a decline in the price-marginal cost margin of a service with a positive margin and the output of that service does not increase sufficiently to offset the margin loss, there will be a reduced ability on the part of the firm to sustain a price index decline, even when efficiency growth is unchanged. This is the reason why, when the output price index is expressed in terms of the cost-weighted TFP measure, an additional term, (Y^R - Y^C), must be included in the equation. This additional term would also need to be included in the price caps formula.

The correct conceptual choice (equation (5) or equation (7)) depends on a comparison of the price/marginal cost relationship in the historical period, from which the productivity offset is drawn, with the relationship expected to prevail in the price caps period. An example drawn from the case of two Canadian telephone companies, Bell Canada and British Columbia Telephone, may clarify the issue. During the 1980s, rates for toll calls exceeded marginal costs and rates for local calls were less than marginal costs. Fuss (1994) demonstrated that this condition, along with the more rapid growth of toll, caused the revenue-weighted TFP growth measure to overestimate substantially efficiency growth. However the revenue-weighted measure might still be the appropriate TFP offset for a price caps plan for these companies. This would occur if the pattern of price, marginal cost relationships were to be continued in the price caps period and there were no significant expected changes in relative growth rates of outputs.

On the other hand, suppose the price caps period represented a period of transition to marginal cost-based pricing; or the price, marginal cost relationships were maintained, but relative output growth rates in the price caps period were expected to differ substantially from the historical period. In that case the conceptually correct productivity offset would be a variable offset which combined the cost-weighted TFP measure with an adjustment term that took into account the changing revenue, cost-weight differentials and the changing relative output growth rates (equation (7)).

While the use of equation (7) would be conceptually correct in the situation described in the last paragraph, it would have several disadvantages from a policy perspective that need to be taken into account. First, marginal cost weights would have to be calculated, and the calculation would likely be controversial. As Norsworthy and Jang (1992) note in the context of US telecommunications, "The use of internal accounting weights to add up the various telecommunication services to a measure of total output is also unlikely to be correct. Accounting practices involve arbitrary methods for allocating fixed costs and major components of variable costs to the various service classes and are unlikely to come reasonably close to the marginal cost weights ..." (page 228).

Second, a price caps formula based on equation (7) would depend on the growth rates of outputs, which creates incentive problems. A LEC would be aware that a lower rate of growth of output for a service which provides a positive margin, or a higher rate of growth for a negative margin service, would result in a lower productivity offset.

A move to a cost-weighted TFP offset would probably be to the advantage of the LECs, in that it would likely result in a lower productivity offset as competition intensifies. This would occur because competitors would target the LECs' high margin services. This targeting would result in reductions in the LECs' price-marginal cost margins and a reduction in the output growth rates for these high margin services. Both impacts would mean that the term (Y^R - Y^C) in equation (7) would decline (or possibly become negative) and the resulting offset would be lower than if the revenue-weighted index were used.

Appendix D

A Comparison of Fisher and Tornquist Indices

The most appropriate way to compare indexing procedures is to utilize the economic theory of index numbers. As developed primarily by Erwin Diewert, the economic theory of index numbers demonstrates that both the Tornquist Index and the Fisher Index belong to the class of superlative indices. A superlative index is an index which corresponds exactly to some second order approximation of an unknown aggregator function which is actually combining the components into an aggregate. From an economic perspective, the Tornquist and Fisher indices only differ because they are exact for different second order approximations. The Tornquist Index is exact for a second order approximating function which is quadratic in the **logarithms** of the components. The Fisher Index is exact for a second order approximating function which is quadratic in the **levels** of the components. Since it is generally unknown which second order approximating function is the better approximation to the true aggregator function, a clear choice is usually not possible. However, for the type of aggregation which is occurring in the Christensen model, it is unlikely to matter. As noted by Diewert (1987, p. 773), in discussing the choice among superlative indexes, "Fortunately, it does not matter very much which of these formulae we choose to use in applications: they will all give the same answer to a reasonably high degree of approximation". The veracity of this quote is demonstrated by the data in table 1 of the text and table D.1 of this appendix.

Since all indexing procedures should be viewed as approximations, there is no conceptual advantage to the fact that the Fisher Price Index is the same approximation numerically whether calculated explicitly or implicitly. The Tornquist indexing procedure gives two distinct approximations to the unknown aggregator function. Either one is as valid as the Fisher approximation. As noted by Diewert, as a practical matter, it is unlikely that the explicit and implicit Tornquist approximations will be significantly different for the data used in the Christensen model. Table D.2 contains a comparison of the two ways of calculating the Tornquist Input Price Index for the Norsworthy data. To three decimal places, the numbers are identical except for 1990, where the numbers differ by one in the third decimal point due to rounding.

Table D.1

A Comparison of Fisher and Tornquist Indices

Norsworthy Data 1985-94

Year	Aggregate Output Index		Aggregate Input Quantity Index		Aggregate Input Price Index	
	Fisher Index	Tornquist Index	Fisher Index	Tornquist Index	Fisher Index	Tornquist Index
1985	1.000	1.000	1.000	1.000	1.000	1.000
1986	1.033	1.033	1.004	1.004	1.055	1.055
1987	1.071	1.071	1.026	1.026	1.052	1.052
1988	1.139	1.139	1.120	1.121	0.985	0.985
1989	1.210	1.210	1.123	1.123	0.992*	0.992
1990	1.289	1.289	1.182	1.182	0.952	0.952
1991	1.350	1.350	1.186	1.187	0.962	0.961
1992	1.402	1.402	1.190	1.190	0.979	0.979
1993	1.470	1.470	1.160	1.160	1.037	1.037
1994	1.555	1.555	1.186	1.186	1.043	1.042

* The corresponding number in Norsworthy's table 5 is 0.982. However, this is a typographical error. The number which appears in Norsworthy's spreadsheet is 0.9925.